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VELOCITY CORRELATIONS IN WEAK TURBULENT SHEAR FLOW

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Deissler,[1] in a study of weak, homogeneous turbulence in a uniform shear flow, reported calculations of several components of the fluctuating velocity correlation $\overline{u_i u_j}$ and the corresponding Fourier transform φ_{ij} for the case of initially isotropic turbulence. Triple correlations in the analysis of weak turbulence were neglected compared to double correlations.

The remaining components of the velocity correlation are presented herein to enable a complete description of the redistribution of turbulent energy $\overline{u_i u_i}$ among its directional components $\overline{u_1^2}$, $\overline{u_2^2}$, and $\overline{u_3^2}$ by the effects of the mean-velocity gradient $a = dU_1/dx_2$ as the turbulence decays. This information supplements recent reports of pressure fluctuations[2] and temperature-gradient effects[3] in shear flows.

Analysis from reference 1 that relates to the present results is outlined as follows for later discussion. In wave-number space, the transform of the two-point velocity correlation $\overline{u_i u_j'}$ satisfies (Eq. (34) of ref. 1)

$$\frac{\partial\varphi_{ij}}{\partial t} = -(\delta_{i1}\varphi_{2j} + \delta_{j1}\varphi_{i2})\frac{dU_1}{dx_2} + \kappa_1\frac{\partial\varphi_{ij}}{\partial\kappa_2}\frac{dU_1}{dx_2} + \left(2\frac{\kappa_1\kappa_i}{\kappa^2}\varphi_{2j} + 2\frac{\kappa_1\kappa_j}{\kappa^2}\varphi_{i2}\right)\frac{dU_1}{dx_2} - 2\nu\kappa^2\varphi_{ij} \qquad (1)$$

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[1] R. G. Deissler, Phys. Fluids, 4, 1187 (1961).

[2] R. G. Deissler, Phys. Fluids, 5, 1124 (1962).

[3] R. G. Deissler, Int. J. Heat Mass Transfer, 6, 257 (1963).

where δ_{ij} is 1 for $i = j$ and 0 for $i \neq j$, and

$$\overline{u_i u_j'} = \int_{-\infty}^{\infty} \varphi_{ij} \exp(i\vec{\kappa} \cdot \vec{r}) d\vec{\kappa}. \tag{2}$$

A scalar dependence on κ is substituted for the vector dependence of φ_{ij} on $\vec{\kappa}$ by integration over a sphere of radius κ,

$$\psi_{ij}(\kappa) = \int_A \varphi_{ij}(\vec{\kappa}) dA(\vec{\kappa}). \tag{3}$$

Similar integration of the terms of Eq. (1) yields

$$\frac{\partial}{\partial t} \psi_{ij} = P_{ij} \frac{dU_1}{dx_2} + T_{ij} \frac{dU_1}{dx_2} + Q_{ij} \frac{dU_1}{dx_2} - 2\nu\kappa^2\psi_{ij} \tag{4}$$

where $P_{ij}(dU_1/dx_2)$ represents production of turbulent energy by the action of the mean velocity gradient, and the last term provides viscous dissipation. Alteration of distributions in wave-number space is effected by $T_{ij}(dU_1/dx_2)$, but no contributions to $\partial\overline{u_i u_j}/\partial t$ result for the case of $\vec{r} = 0$ that is considered here. A zero pressure-force term occurs in the contracted equation; that is, $Q_{ii}(dU_1/dx_2) = 0$ where summation is indicated by the repeated index. Thus, the pressure-force terms Q_{ij} can be interpreted as exchanging energy between the directional components of energy but contributing nothing to $\partial\psi_{ii}/\partial t$.

A solution to Eq. (1), in addition to φ_{22}, φ_{12}, and φ_{ii} from reference 1, that satisfies an isotropic initial condition,

$$\left(\varphi_{ij}\right)_0 = \left(J_0/12\pi^2\right)\left(\kappa^2\delta_{ij} - \kappa_i\kappa_j\right), \tag{5}$$

is

$$
\begin{aligned}
\varphi_{11} = {} & \frac{J_0\left\{\kappa_1^2 + \left[\kappa_2 + a\kappa_1(t - t_0)\right]^2 + \kappa_3^2\right\}^2}{12\pi^2} \\
& \times \exp\left\{-2\nu(t - t_0)\left[\kappa^2 + \frac{1}{3}\kappa_1^2 a^2 (t - t_0)^2 + a\kappa_1\kappa_2(t - t_0)\right]\right\} \\
& \times \left\{\frac{\kappa_1^2}{\kappa_1^2 + \kappa_3^2}\left(\frac{\kappa_2^2}{\kappa^4} - \left\{\frac{\kappa_2 + a\kappa_1(t - t_0)}{\kappa_1^2 + \left[\kappa_2 + a\kappa_1(t - t_0)\right]^2 + \kappa_3^2}\right\}^2\right)\right. \\
& + \frac{\left[\kappa_2 + a\kappa_1(t - t_0)\right]^2 + \kappa_3^2}{\left\{\kappa_1^2 + \left[\kappa_2 + a\kappa_1(t - t_0)\right]^2 + \kappa_3^2\right\}^2} - \frac{2\kappa_2\kappa_3^2}{\left(\kappa_1^2 + \kappa_3^2\right)^{3/2}\kappa^2} \\
& \times \left[\tan^{-1}\frac{\kappa_2}{\left(\kappa_1^2 + \kappa_3^2\right)^{1/2}} - \tan^{-1}\frac{\kappa_2 + a\kappa_1(t - t_0)}{\left(\kappa_1^2 + \kappa_3^2\right)^{1/2}}\right] \\
& \left. + \frac{\kappa_3^4}{\left(\kappa_1^2 + \kappa_3^2\right)^2\kappa_1^2}\left[\tan^{-1}\frac{\kappa_2}{\left(\kappa_1^2 + \kappa_3^2\right)^{1/2}} - \tan^{-1}\frac{\kappa_2 + a\kappa_1(t - t_0)}{\left(\kappa_1^2 + \kappa_3^2\right)^{1/2}}\right]^2\right\}.
\end{aligned}
\tag{6}
$$

An explicit form for φ_{33} is not shown here since it can be obtained by subtraction as

$$\varphi_{33} = \varphi_{ii} - \varphi_{11} - \varphi_{22}. \tag{7}$$

Solutions for φ_{13} and φ_{23}, although nonzero, are omitted since the corresponding velocity correlations $\overline{u_1u_3}$ and $\overline{u_2u_3}$ are zero, which seems physically reasonable from the lack of velocity gradients in the x_1, x_3 and x_2, x_3 planes. It is interesting that ψ_{13} and ψ_{23} are also zero so that a zero average of φ_{13} and φ_{23} is obtained over every sphere of radius κ.

Dimensionless spectra of $\frac{1}{2}\,\overline{u_1^2}$ and $\frac{1}{2}\,\overline{u_3^2}$ are displayed in Figs. 1 and 2 in a temporally invariant representation. By far the largest component of the energy-spectrum function $\frac{1}{2}\,\psi_{ii} = E$, as shown in Fig. 1, is formed by $\frac{1}{2}\,\psi_{11}$ at large velocity gradients. Energy production, the apparent cause of this effect, occurs only in the ψ_{11} equation [second and third terms in Eq. (1)] and outweighs the transfer of energy between components by the pressure forces. An interpretation of these terms as <u>production</u> of energy is aided by changing to negative the sign of the spectrum of $\overline{u_1u_2}$ that appears in Fig. 6 of reference 1 so that agreement with Fig. 9 of reference 1 and the usual convention is attained.

The reshaping of the spectral distribution of $\overline{u_3^2}$ at large velocity gradients is shown in Fig. 2 by the dashed-line distribution corresponding to zero velocity gradient but normalized to the peak of a large velocity-gradient distribution.

On Fig. 3, Q_{11} is shown to be always negative, indicating that pressure forces transfer energy out of the spectrum of $\overline{u_1^2}$ at all wave numbers; conversely, Q_{33} is positive, indicating that energy is transferred into the spectrum $\overline{u_3^2}$. On the other hand, Q_{22} is negative at high wave numbers but positive at low wave numbers in varying proportions, so that the net contribution to $\partial\overline{u_2^2}/\partial t$ is positive at small velocity gradients but is negative at large gradients. Most of the energy transfer at high velocity gradients by the pressure terms is from the spectrum of $\overline{u_1^2}$ into the spectrum of $\overline{u_3^2}$; smaller transfer is effected by Q_{22}.

Curves of $\overline{u_1^2}$ and $\overline{u_3^2}$ that are shown in Fig. 4 were obtained by integrating the spectral distributions of Figs. 1 and 2. A curve for $\overline{u_2^2}$ is reproduced from reference 1. As anticipated from the spectral results, $\overline{u_1^2}$ as a fraction of $\overline{u_i u_i}$ rises rapidly with increasing velocity gradient. From the spectral distributions, as well as Fig. 4, pressure forces are shown to transfer more energy into $\overline{u_3^2}$ than $\overline{u_2^2}$ at high velocity gradients, but a reverse effect at low gradients is also shown in Fig. 4 where $\overline{u_2^2}$ is greater than $\overline{u_3^2}$.

The shear correlation coefficient $\overline{u_1 u_2}\Big/\left(\overline{u_1^2}\,\overline{u_2^2}\right)^{1/2}$ that is displayed in Fig. 4 is analogous to the heat-transfer correlation coefficient that was introduced by Corrsin[4] and, for shear flows, was calculated by Deissler.[3] After a rapid rise from the expected zero value for no-velocity gradient, the shear coefficient is remarkably constant over a wide range of gradients.

[4]S. Corrsin, J. Appl. Phys., <u>23</u>, 113 (1952).

CAPTIONS

Fig. 1 - Dimensionless spectra of $\frac{1}{2}\overline{u_1^2}$ and $\overline{u_i u_i}$.

Fig. 2 - Dimensionless spectra of $\frac{1}{2}\overline{u_3^2}$. — — — curve for $dU_1/dx_2 = 0$ normalized at the peak to the curve for $dU_1/dx_2 = 20$.

Fig. 3 - Spectral pressure-force terms from Eq. (4).

Fig. 4 - Velocity correlation ratios as a function of dimensionless velocity gradient.

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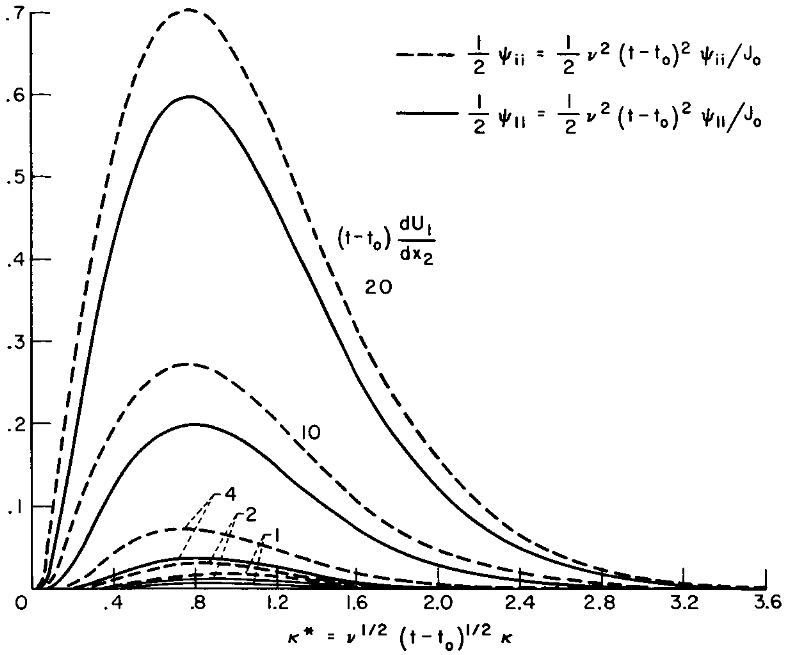

Fig. 1 Dimensionless spectra of $\frac{1}{2}\overline{u_1^2}$ and $\frac{1}{2}\overline{u_i u_i}$.

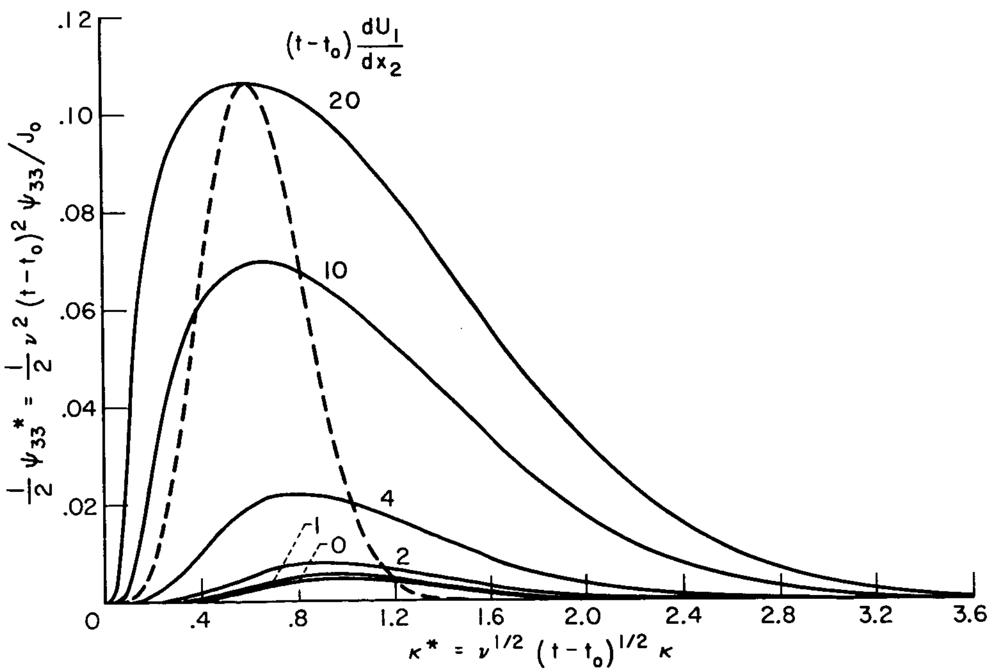

Fig. 2 Dimensionless spectra of $\frac{1}{2}\overline{u_3^2}$, ——— curve for $dU/dx_2 = 0$ normalized at the peak to the curve for $dU_1/dx_2 = 20$.

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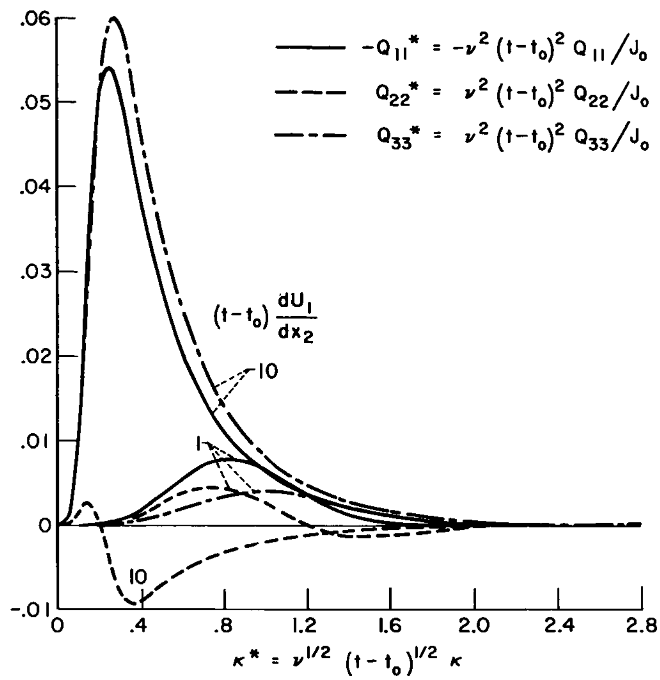

Fig. 3 Spectral pressure-force terms from Eq. (4).

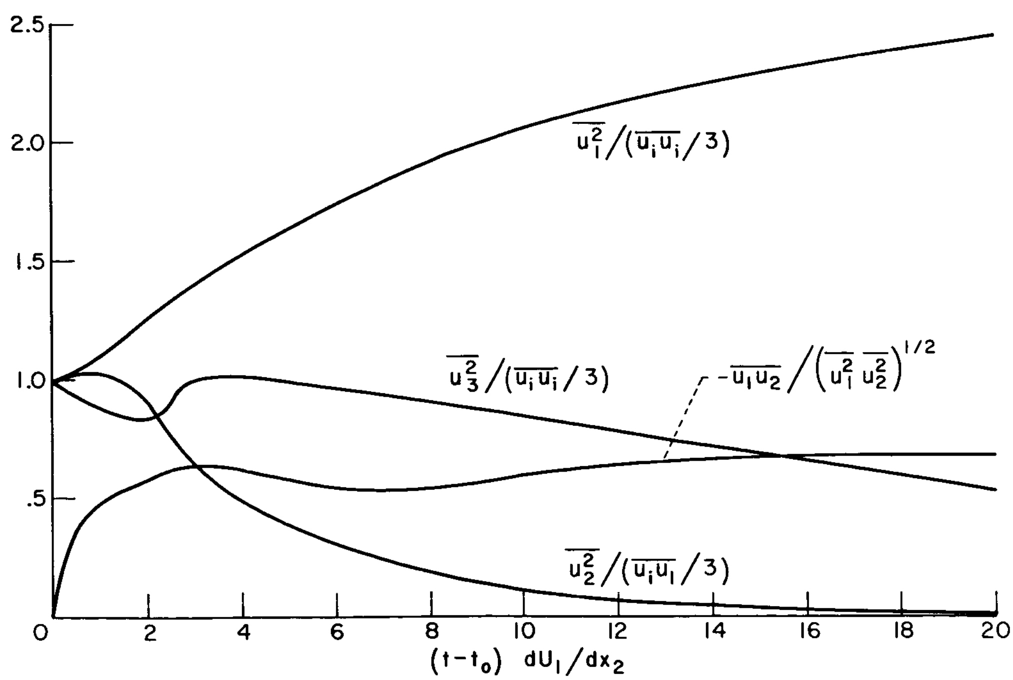

Fig. 4 Velocity correlation ratios as a function of dimensionless velocity gradient.